CHAPTER 7
SEQUENCES AND SERIES

Big IDEAS:
1) Analyze sequences
2) Find sums of series
3) Use recursive rules

<table>
<thead>
<tr>
<th>Section</th>
<th>7-1 Define and Use Sequences and Series</th>
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<tr>
<td>Essential Question</td>
<td>How can you write an expression for sums?</td>
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Key Vocabulary

<table>
<thead>
<tr>
<th>Sequence (Pattern)</th>
<th>A function whose domain is a set of _________ integers. The domain starts with _____ unless stated otherwise.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1, 4, 9, 16, 25, 36</td>
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<tr>
<td>Terms</td>
<td>The values in the __________.</td>
</tr>
<tr>
<td>Finite Sequence</td>
<td>Sequence containing a __________ number of terms.</td>
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<td></td>
<td>Domain: _________________</td>
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<td></td>
<td>Range: ________________</td>
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<td></td>
<td>Rule : _________________</td>
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<tr>
<td>Infinite Sequence</td>
<td>A sequence that __________ without stopping.</td>
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<tr>
<td>Series</td>
<td>Expression resulting from __________ the terms of a sequence together.</td>
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<td></td>
<td>[ \sum_{i=1}^{20} 5i ]</td>
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<tr>
<td>Summation Notation (Sigma Notation)</td>
<td>Used to write a __________.</td>
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Ex 1) Write the first five terms of each sequence.
   a. \[ a_n = 5n - 1 \]  
   b. \[ f(n) = 2^n - 3 \]
Ex 2) Describe the pattern, write the next term, and write a rule for the $n$th term of the sequence.

a. 2, 5, 10, 17,...

b. $-4, -8, -12, -16,...$

Ex 3) At the beginning of the summer, you walk a mile in 9 minutes. At the end of the first week you walk a mile in 5 fewer seconds. At the end of the second week, you decrease your time by another 10 seconds. At the end of the third week, you decrease your time by 20 more seconds, and at the end of the forth week, you decrease your time by another 40 seconds. Write a rule for the number of seconds you decrease your time by each week. Then graph the sequence for the first five weeks.

Ex 4) Write the series using summation notation.

a. $1 + 8 + 27 + 64 + ... + 729$

b. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$
### Formulas for Special Series

<table>
<thead>
<tr>
<th>Sum of $n$ terms of 1</th>
<th>Sum of first $n$ positive integers</th>
<th>Sum of squares of first $n$ positive integers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i=1}^{n} 1 = n$</td>
<td>$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$</td>
<td>$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$</td>
</tr>
</tbody>
</table>

**Ex 5)** Find the sum of the series.

a. $\sum_{k=3}^{5} (3k^2 - 7)$  

b. $\sum_{i=1}^{34} 1$

**Ex 6)** You work in a grocery store and are stacking apples in the shape of a pyramid with 8 layers.

a. Write a rule for the number of apples in each layer.

b. Use a formula to find how many apples are in the stack.
Section: 7-2 Analyze Arithmetic Sequences and Series

Essential Question
How can you tell that a sequence is arithmetic?

Key Vocabulary

<table>
<thead>
<tr>
<th>ARITHMETIC SEQUENCE</th>
<th>A sequence in which the difference of consecutive terms is ________.</th>
<th>17, 14, 11, 8, 5,...</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON DIFFERENCE ((d))</td>
<td>The constant ____________ in an arithmetic sequence.</td>
<td></td>
</tr>
<tr>
<td>ARITHMETIC SERIES</td>
<td>The expression formed by ____________ the terms of an arithmetic sequence.</td>
<td></td>
</tr>
</tbody>
</table>

**Rule for Arithmetic Sequence**

The \(n\)th term of an arithmetic sequence with first term \(a_1\) and common difference \(d\) is given by:

\[ a_n = a_1 + (n-1)d \]

**Ex 1)** Tell whether the sequence is arithmetic.

a. 7, 13, 19, 25,...

b. -8, -4, 0, 6, 12,...

**Ex 2)** Write a rule for the \(n\)th term of the sequence. Then find \(a_{20}\).

a. -7, -10, -13, -16,...

b. 59, 68, 77, 86,...
Ex 3) One term of an arithmetic sequence is \( a_{27} = 263 \). The common difference is \( d = 11 \).

a. Write a rule for the \( n \)th term.  
b. Graph the first 6 terms of the sequence.

Ex 4) Two terms of an arithmetic sequence are \( a_{10} = 148 \) and \( a_{44} = 556 \). Find a rule for the \( n \)th term.

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**Sum of a Finite Arithmetic Series**

The sum of the first \( n \) terms of an arithmetic series is:

\[
S_n = n \left( \frac{a_1 + a_n}{2} \right)
\]

*** \( S_n \) is the mean (average) of the first and last terms, multiplied by the number of terms.***

Ex 5) What is the sum of the arithmetic series \( \sum_{i=1}^{28} (-2 + 4i) \)?
Section: 7-3 Analyze Geometric Sequences and Series

Essential Question
How can you tell that a sequence is geometric?

Key Vocabulary

<table>
<thead>
<tr>
<th><strong>Geometric Sequence</strong></th>
<th>A sequence in which the ______ of any term to the previous term is constant.</th>
<th>625, 125, 25, 5, 1,...</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Ratio</strong> <em>(r)</em></td>
<td>The constant ______ in a geometric sequence.</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric Series</strong></td>
<td>The expression formed by ________ the terms of a geometric sequence.</td>
<td></td>
</tr>
</tbody>
</table>

Rule for Geometric Sequence

The *n*th term of a geometric sequence with first term *a*₁ and common ratio *r* is given by:

\[ a_n = a_1 r^{n-1} \]

Ex 1) Tell whether the sequence is geometric.
   a. 2, 6, 18, 54, 162,...
   b. 2, 7, 12, 17, 22,...

Ex 2) Write a rule for the *n*th term of the sequence. Then find *a*₆.
   a. 3, 12, 48, 192,...
   b. 36, −12, 4, −\(\frac{4}{3}\),...
Ex 3) One term of a geometric sequence is \(a_5 = 54\). The common ratio is \(r = 3\).

a. Write a rule for the \(n^{th}\) term.

Ex 4) Two terms of a geometric sequence are \(a_3 = 45\) and \(a_5 = 405\). Find a rule for the \(n^{th}\) term.

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**Sum of a Finite Geometric Series**

The sum of the first \(n\) terms of a geometric series with common ratio \(r \neq 1\) is:

\[
S_n = a_1 \left( \frac{1-r^n}{1-r} \right)
\]

Ex 5) Find the sum of the geometric series \(\sum_{i=1}^{6} 4 \left( \frac{3}{2} \right)^{i-1}\)?
Ex 6) In 1999 about 586 thousand used cars were sold in Maryland. From 1999 through 2004, the number of used cars sold increased by about 3.8% per year.

a. Write a rule for the total number of used cars sold $a_n$ (in thousands) in terms of the year. Let $n = 1$ represent 1999.

b. What was the total number of used cars sold in Maryland from 1999 to 2004?

Ex 7) A regional soccer tournament has 64 participating teams. In the first round of the tournament, 32 games are played. In each successive round, the number of games played decreases by a factor of one half.

a. Find a rule for the number of games played in the $n$th round. For what values of $n$ does your rule make sense?

b. Find the total number of games played in the regional soccer tournament.
Section: 7-4 Find Sums of Infinite Geometric Series

Essential Question
When does an infinite geometric series have a sum?

Complete p.459 Investigating an Infinite Geometric Series

# of Pieces

Compared Area

Key Vocabulary

Partial Sum- the sum $S_n$ of the first $n$ terms of an infinite series

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term $a_1$ and common ratio $r$ is given by

$$S = \frac{a_1}{1-r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has _____ sum.

Ex 1) Consider the infinite geometric series $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots$

Find and graph the partial sums $S_n$ for $n = 1, 2, 3, 4,$ and $5$. Then describe what happens to $S_n$ as $n$ increases.
Ex 2) Find the sum of the infinite geometric series.

a. \[ \sum_{i=1}^{\infty} \left( \frac{1}{4} \right)^{i-1} \]  

b. \[ \frac{1}{2} - \frac{1}{4} - \frac{1}{8} + \ldots \]

Ex 3) What is the sum of the infinite series \[ 1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \ldots \]?

Ex 4) A rubber ball is dropped from a height of 60 feet. Each bounce takes it to \( \frac{2}{3} \) of its previous height.

What is the total vertical distance the ball travels?

Ex 5) Write 5.146146146... as a fraction in lowest terms.
Key Vocabulary

Explicit Rule - gives \(a_n\) as a function of the term’s ______________ number \(n\) in the sequence.

Recursive Rule - gives the ______________ term or terms of a sequence and then a ______________ equation that tells how the \(n\)th term \(a_n\) is related to one or more preceding terms.

Iteration - the repeated ______________ of a function with itself

\[x_i = f(x_0), \quad x_2 = f(x_1), \ldots\]

**RECURSIVE EQUATIONS FOR ARITHMETIC AND GEOMETRIC SEQUENCES**

**Arithmetic Sequence**

\[a_n = a_{n-1} + d\] where \(a_{n-1}\) is the ______________ term and \(d\) is the common difference.

**Geometric Sequence**

\[a_n = r \cdot a_{n-1}\] where \(a_{n-1}\) is the ______________ term and \(r\) is the common ratio.

Ex 1) Write the first six terms of the sequence.

\[
\begin{align*}
\text{a.} \quad & a_1 = -2, \quad a_n = a_{n-1} + 3 \\
\text{b.} \quad & a_1 = 32, \quad a_n = \frac{1}{2} a_{n-1}
\end{align*}
\]
Ex 2) Write a recursive rule for the sequence.

a. $100, 40, 16, \frac{32}{5}, \frac{64}{25}, \ldots$

b. $8, 28, 48, 68, 88, \ldots$

c. $-10, 8, 18, 10, -8, -18, \ldots$

d. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

Ex 3) A mosquito population in a controlled laboratory condition is estimated to be about 500. Each day an additional 100 mosquitoes are hatched. The population also declines by 85% every day from a pesticide and other natural causes.

a. Write a recursive rule for the number $a_n$ of mosquitoes at the start of the $n$th day.

b. Find the number of mosquitoes after the 5th day.

c. Describe what happens to the number of mosquitoes over time.

Ex 4) Find the first three iterates $x_1, x_2$, and $x_3$ of the function $f(x) = 5x - 3$ for an initial value of $x_0 = 1$. 