Grade Level Summary

This course is designed to follow the AP statistics curriculum (A complete course description can be found at http://media.collegeboard.com/digitalServices/pdf/ap/ap-statistics-course-description.pdf). The course will provide an introduction to statistical methods and data analyses that are common to a first level collegiate course. It will address topics in both descriptive and inferential statistics. Topics will fall under one of four major headings: (1) Exploring Data – Observing patterns and departures from patterns; (2) Planning a Study – Deciding what and how to measure; (3) Anticipating patterns – Producing models using probability theory and simulation; and (4) Statistical Inference – Confirming models. A teacher recommendation is required along with the completion of a Trigonometry/Advanced Mathematics course.

Grade Level Units

Unit 1: The Role of Statistics and the Data Analysis Process
- Unit 2: Collecting Data Sensibly
- Unit 3: Graphical Methods for Describing Data
- Unit 4: Numerical Methods for Describing Data
- Unit 5: Summarizing Bivariate Data
- Unit 6: Probability
- Unit 7: Random Variables and Probability Distributions
- Unit 8: Sampling Variability and Sampling Distributions
- Unit 9: Estimation Using a Single Sample
- Unit 10: Hypothesis Testing Using a Single Sample
- Unit 11: Comparing Two Populations or Treatments
- Unit 12: The Analysis of Categorical Data and Goodness-of-Fit Tests
- Unit 13: Simple Linear Regression and Correlation: Inferential Methods
- Unit 14: Multiple Regression Analysis
- Unit 15: Analysis of Variance

Unit Title

Unit 1: The Role of Statistics and the Data Analysis Process

Unit Overview

An understanding of statistics equips us to make intelligent judgments and informed decisions. This unit provides an overview of the importance of having a basic understanding of statistics. The nature and role of variability is discussed and the general process for performing data analysis is outlined. The unit also provides an introduction of some simple graphical displays including frequency distributions, bar charts and dotplots.

Unit Essential Questions

1. Why is it important to study and understand statistics?
2. Why is it important to understand variability?
3. What is statistics?
4. What are the six steps in the data analysis process?
5. What is the difference between categorical and numerical data?
6. What simple graphical displays can be used to display categorical and numerical data and how are they constructed?

Key Understandings

1. Explain how the study of statistics plays a role in being an informed consumer and making informed judgments
2. Explain how the study of variability allows one to distinguish between usual and unusual values
3. Define and describe the two major branches of statistics – descriptive statistics and inferential statistics
4. Outline the six steps which are typically followed when analyzing data
Focus Standards Addressed in the Unit

| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.4.HS.B.1 | Summarize, represent, and interpret data on a single count or measurement variable. |

Important Standards Addressed in the Unit

| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.4.HS.B.1 | Summarize, represent, and interpret data on a single count or measurement variable. |

Misconceptions

1. An understanding of statistics is only important for individuals entering math related fields of study.
2. Descriptive and inferential statistics both involve drawing conclusions from data.
3. Any type of graphical display can be used to represent any type of data.

Proper Conceptions

1. The average person must understand statistics in order to be an informed consumer and to make informed judgments.
2. Descriptive statistics and inferential statistics refer to two different branches of statistics. Descriptive statistics involves ways to describe and display data. Inferential statistics involves the processes used to make generalizations about a population.
3. Different graphical displays are used for the different types of data (categorical and numerical).

Concepts

- Reasons to study statistics
- Nature and role of variability
- The data analysis process
- Types of data
- Simple graphical displays

Competencies

- Explain why it is important to have an understanding of statistics
- Outline the six steps in the data analysis process
- Distinguish between descriptive and inferential statistics
- Distinguish between a population and a sample
- Distinguish between categorical and numerical data
- Distinguish between discrete and continuous numerical data
- Distinguish between univariate, bivariate, and multivariate data
- Construct and interpret frequency distributions
- Construct and interpret bar charts
- Construct and interpret dotplots

Vocabulary

- Population
- Sample
- Descriptive statistics
- Inferential statistics
- Categorical data
- Numerical data
- Discrete numerical data
- Continuous numerical data
- Univariate data
- Bivariate data
- Multivariate data
- Frequency distribution
- Bar chart
- Dot plot

Assessments

Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit.
Unit Notebook – Students’ notebooks will be checked at the end of each instructional unit to reinforce organizational skills
Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment

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**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction*

3a – An assignment sheet for the unit will communicate the timeline for the unit as well as the assignments and assessments to be completed.

3b – Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews.

3c – Students will be required to complete various activities and assignments, many of which will involve collaborative efforts. Students will also be required to take notes using a skeletal note packet which will serve as a study guide.

3d – Students will evaluate their own understanding during homework reviews. Students will be assessed informally on a daily basis through student/teacher interactions. Summative assessments will include a test and a project which will provide students with feedback concerning their understanding of the concepts.

3e – Pacing of lessons may be adjusted and additional examples may be given as necessary to enhance student understanding.

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**Differentiation:**

- Graphic organizers may be provided.
- Multiple concrete examples of concepts will be provided.
- Extended time may be permitted for completing projects, quizzes, or tests.
- Lessons will be presented using both visual and auditory means of communication.

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**Interdisciplinary Connections:**

- Biology
- College Life
- Demography and Population Characteristics
- Environmental Science
- Leisure and Popular Culture
- Psychology, Sociology and Social Issues
- Transportation

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**Additional Resources (May include but are not limited to the following):**

- Textbook Ancillary Materials
- College Board AP Course Guidelines
- Released AP Test Questions
- AP Statistics Test Preparation Workbooks
- [www.collegeboard.org](http://www.collegeboard.org)

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**Created By:**

Thomas A. Seltzer
Unit Title
Unit 2: Collecting Data Sensibly

Unit Overview
In order to make informed decisions through statistical studies, it is important to collect data in a reliable manner and analyze it appropriately. This unit will address two types of statistical studies; observational studies and experimental studies. It will also discuss two popular methods for collecting reliable data; sampling and experimentation.

Unit Essential Questions
1. What is the difference between an observational study and an experiment?
2. What are confounding variables?
3. What are the methods used for sampling?
4. What are the different types of sampling bias?
5. What are the four key concepts in experimental design?
6. What is meant by a single-blind and a double-blind experiment?

Key Understandings
1. Explain the difference between an observational study and an experiment
2. Explain various sampling methods
3. Explain the various forms of bias
4. Describe what is meant by confounding variables
5. Identify the explanatory and response variables in a simple comparative experiment
6. Outline the key concepts in experimental design

Focus Standards Addressed in the Unit

CC.2.4.HS.B.4 Recognize and evaluate random processes underlying statistical experiments.

CC.2.4.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
**Important Standards Addressed in the Unit**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC.2.4.HS.B.1</td>
<td>Summarize, represent, and interpret data on a single count or measurement variable.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.2</td>
<td>Summarize, represent, and interpret data on two categorical and quantitative variables.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.4</td>
<td>Recognize and evaluate random processes underlying statistical experiments.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.5</td>
<td>Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.</td>
</tr>
</tbody>
</table>

**Misconceptions**

1. The method by which data is collected for statistical analyses is not important.
2. Observational studies can be used to draw a cause-and-effect conclusion.
3. You may always generalize from a sample to a population.
4. All sampling methods provide reliable data about the population of interest.

**Proper Conceptions**

1. It is important to think about the research objectives prior to collecting data and then develop a plan for collecting the data in a reliable manner.
2. Observational studies cannot be used to draw a cause-and-effect conclusion. However, a well-designed experiment can result in data that provide evidence for a cause-and-effect relationship.
3. You may only generalize from sample to population when the sample is representative of the population. That is, the sample is a random sample from the population of interest and has no major potential sources of bias.
4. Voluntary response and convenience sampling are usually not representative of a population and should not be used to generalize conclusions to a larger population.

**Concepts**

- Statistical Studies: observation and experimentation
- Drawing conclusions from statistical studies
- Sampling methods
- Bias in sampling
- Key concepts in experimental design
- Structuring experimental designs
- Use of placebos
- Single- and double-blind experiments
- Designing Surveys

**Competencies**

- Compare and contrast observational and experimental studies
- Perform both observational and experimental studies and know when it is appropriate to generalize the results to a population - Also know when it is appropriate to draw a cause-and-effect conclusion.
- Identify, describe, and perform various sampling methods
- Identify the different forms of bias in sampling
- Apply the 4 key concepts in experimental design
- Use a diagram to represent an experimental design
- Understand the effect of using a placebo
- Explain the difference between a single-blind and a double-blind experiment
- State considerations for constructing surveys
- Interpret and communicate results from statistical analyses

**Vocabulary**

- Observational study
- Simple random sample
- Stratified sampling
- Cluster sampling
- 1 in k Systematic sampling
- Confounding variable
- Measurement or response bias
- Selection bias
- Nonresponse bias
- Experiment
- Treatments
- Extraneous variable
- Direct control
- Blocking
- Random assignment
- Replication
- Placebo treatment
- Control group
- Single-blind experiment
- Double-blind experiment
Assessments
Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Quizzes – Quizzes will be given throughout the unit to evaluate students’ understanding of the material.
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit.
Unit Notebook – Students’ notebooks will be checked at the end of each instructional unit to reinforce organizational skills
Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment

Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction

3a – An assignment sheet for the unit will communicate the timeline for the unit as well as the assignments and assessments to be completed.
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3e – Pacing of lessons may be adjusted and additional examples may be given as necessary to enhance student understanding.

Differentiation:
• Graphic organizers may be provided.
• Multiple concrete examples of concepts will be provided.
• Extended time may be permitted for completing projects, quizzes, or tests.
• Lessons will be presented using both visual and auditory means of communication.

Interdisciplinary Connections:
Education and Child Development
Food Science
Leisure and Popular Culture
Medical Science
Psychology, Sociology and Social Issues
Public Health and Safety
Surveys and Opinion Polls
Transportation

Additional Resources (May include but are not limited to the following):
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
AP Statistics Test Preparation Workbooks
www.collegeboard.org

Created By:
Thomas A. Seltzer
Unit Title
Unit 3: Graphical Methods for Describing Data

Unit Overview
Many questions concerning data can be easily answered if the data is organized and displayed in a sensible manner. This unit introduces several methods for organizing and describing both categorical and numerical data using tables and graphs. Methods for displaying categorical data include comparative bar charts, pie graphs and segmented bar charts. Methods for displaying numerical data include stem and leaf displays, frequency distributions, histograms, cumulative relative frequency plots, scatterplots, and time series plots.

Unit Essential Questions
1. When should a comparative bar chart, a pie chart or a segmented bar chart be used to display data and how are each of these constructed?
2. When should stem-and-leaf displays, frequency distributions, histograms, cumulative relative frequency plots, scatterplots and time series plots be used to display data and how are each of these constructed?
3. When interpreting each of the displays in this unit, what should you look for?

Key Understandings
1. Use correct methods for constructing categorical displays and know when it is appropriate to use each type of display (Correct procedures for constructing comparative bar charts, pie charts and segmented bar charts)
2. Use correct methods for constructing numerical displays and know when it is appropriate to use each type of display (Correct procedures for constructing stem-and-leaf displays, frequency distributions, histograms, cumulative relative frequency plots, scatterplots and time series plots)
3. Know what to look for in graphical displays

Focus Standards Addressed in the Unit
CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.
CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.
## Important Standards Addressed in the Unit

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
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<tbody>
<tr>
<td>CC.2.1.HS.F.3</td>
<td>Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.</td>
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<tr>
<td>CC.2.4.HS.B.1</td>
<td>Summarize, represent, and interpret data on a single count or measurement variable.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.2</td>
<td>Summarize, represent, and interpret data on two categorical and quantitative variables.</td>
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## Misconceptions

1. It is not important to draw graphs to scale.
2. It is okay to show a break-in-scale on all types of graphs with scaled axes.
3. Observations for a time series plot that are made at different time intervals should be spaced evenly across the x-axis.
4. A strong pattern in a scatterplot means there is a cause-and-effect relationship between the two variables.
5. It is okay to distort graphical displays to make them more visually interesting.

## Proper Conceptions

1. Areas should be proportional to frequency, relative frequency or magnitude of the number being represented.
2. In bar charts and histograms, the vertical axis should never be broken.
3. If observations over time are not made at a regular time interval, care must be taken to construct a time series plot so that the observations are plotted in the correct location.
4. A strong pattern in a scatterplot does not imply a cause-and-effect relationship.
5. Graphical displays should make a “right” first impression. The reader should be able to get a correct impression from the display with just a quick glance.

## Concepts

- Comparative bar charts
- Pie charts
- Segmented bar graphs
- Stem-and-leaf displays
- Frequency distributions
- Histograms
- Density
- Cumulative relative frequencies
- Cumulative relative frequency plots
- Scatterplot
- Time-series plot

## Competencies

- Correctly construct appropriate graphical displays for categorical data: comparative bar charts, pie graphs, and segmented bar charts
- Correctly construct appropriate graphical displays for numerical data: stem-and-leaf displays, frequency distributions, histograms, cumulative relative frequency plots, scatterplots and time series plots
- Interpret and communicate results from statistical analyses

## Vocabulary

- Frequency distribution
- Comparative bar chart
- Pie chart
- Segmented bar graph
- Stem-and-leaf display
- Histogram
- Density
- Histogram shapes
- Cumulative relative Frequency plot
- Scatterplot
- Time-series plot

## Assessments

Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Quizzes – Quizzes will be given throughout the unit to evaluate students’ understanding of the material.
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit.
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Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction

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Differentiation:

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• Lessons will be presented using both visual and auditory means of communication.

Interdisciplinary Connections:

Agriculture
Biology
Business and Economics
College Life
Demography and Population Characteristics
Education and Child Development
Environmental Science
Sports
Transportation

Additional Resources (May include but are not limited to the following):

Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
AP Statistics Test Preparation Workbooks
www.collegeboard.org

Created By:
Thomas A. Seltzer
Unit Title
Unit 4: Numerical Methods for Describing Data

Unit Overview
When describing numerical data, it is useful to provide information concerning the center and spread of the data. This unit focuses on the different measures of center and spread that can be used to describe a numerical data set. Specifically this unit will discuss mean and median as a measure of center as well as variance and standard deviation as measures of spread. The use of 5-number summaries and the construction of boxplots is discussed as a useful means for obtaining information about the center, spread and symmetry or skewness of a data set. This unit will also present a discussion of Chebyshev’s Rule the Empirical Rule and z-scores as they relate to the interpretation of center and variability.

Unit Essential Questions
1. How are the mean and median calculated and when are they used?
2. How are the variance and standard deviation calculated and when are they used?
3. What is a trimmed mean and how is it calculated?
4. What is a boxplot and how is it constructed?
5. What is the IQR and what information does it provide?
6. What is a sample proportion?
7. What is Chebyshev’s Rule
8. What is the Empirical Rule
9. What is a z-score

Key Understandings
1. Find the mean, median, variance, standard deviation, and 5-number summary of a data set and know when it is appropriate to use each to describe the data set
2. Construct and interpret a boxplot (and show outliers)
3. Calculate trimmed means, sample proportions, and IQR
4. Correctly apply Chebyshev’s Rule, the Empirical Rule and z-scores when interpreting center and variability of a data set

Focus Standards Addressed in the Unit

CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.
CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

Important Standards Addressed in the Unit

CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

Misconceptions
1. The mean and the median give enough information about the behavior of a data set.
2. Distributions with different shapes will have different means and standard deviations.
3. It does not matter which measures of center and spread are used to represent a data set.
4. The Empirical Rule can be applied to all distributions.
5. Outliers do not affect the interpretation of a data set.

Proper Conceptions
1. The mean and median only give us what is considered to be a “typical value” of a data set. Additional information concerning variability of the data and the shape of the distribution is necessary to see the behavior of a variable.
2. Distributions with different shapes can have the same mean and standard deviation.
3. Mean and standard deviation should be used when the data set is approximately normal and has no outliers. The median and IQR should be used if the data set is skewed or if it has outliers.
4. Chebyshev’s rule applies to all distributions. The Empirical Rule can only be applied when the distribution is normal or approximately normal.
5. Outliers often provide important information about the variable that is being studied.

Concepts
- Mean
- Median
- Variance
- Standard deviation
- Interquartile range
- Boxplot
- Chebyshev’s rule
- Empirical rule
- Z-scores

Competencies
- Calculate and interpret measures of center and spread including the mean, median, variance, standard deviation, and interquartile range
- Construct and correctly interpret a boxplot
- Correctly interpret and apply Chebyshev’s rule and the Empirical Rule
- Compute and interpret z-scores

Vocabulary
- Sample mean
- Population mean
- Sample median
- Trimmed mean
- Deviations from the mean:
  - Sample variance
  - Sample standard deviation

Assessments
Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Quizzes – Quizzes will be given throughout the unit to evaluate students’ understanding of the material
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit
Unit Notebook – Students’ notebooks will be checked at the end of each instructional unit to reinforce organizational skills
Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment

Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction

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Pacing of lessons may be adjusted and additional examples may be given as necessary to enhance student understanding.

**Differentiation:**
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**Interdisciplinary Connections:**
- Biology
- Business and Economics
- College Life
- Demography and Population Characteristics
- Education and Child Development
- Marketing and Consumer Behavior
- Psychology, Sociology and Social Issues
- Sports
- Transportation

**Additional Resources (May include but are not limited to the following):**
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**Created By:**
- Thomas A. Seltzer
Unit Title
Unit 5: Summarizing Bivariate Data

Unit Overview
This unit introduces methods used for describing relationships between two numerical variables and for assessing the strength of the relationship between them. The methods described in this unit allow us to answer questions regarding the relationship between the two variables while providing a quantitative description of the relationship.

Unit Essential Questions
1. What is correlation and how is it interpreted?
2. What is the population correlation coefficient?
3. What is the LSRL and how is it found?
4. How is the fit of a line assessed?
5. What is the coefficient of determination and how is it interpreted?
6. How is the standard deviation about the LSRL calculated and interpreted?
7. What is meant by a transformation?

Key Understandings
1. Calculate, interpret and recognize the properties of Pearson’s Sample Correlation Coefficient
2. Know the difference between r and \( \rho \)
3. Calculate and interpret the LSRL for bivariate data
4. Calculate and interpret predicted values and residual values for a LSRL
5. Calculate and interpret the coefficient of determination
6. Calculate and interpret the standard deviation about the LSRL
7. Transform a nonlinear function

Focus Standards Addressed in the Unit

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
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<tbody>
<tr>
<td>CC.2.4.HS.B.2</td>
<td>Summarize, represent, and interpret data on two categorical and quantitative variables.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.3</td>
<td>Analyze linear models to make interpretations based on the data.</td>
</tr>
</tbody>
</table>
## Important Standards Addressed in the Unit

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<tr>
<td>CC.2.1.HS.F.3</td>
<td>Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.</td>
</tr>
<tr>
<td>CC.2.2.HS.D.7</td>
<td>Create and graph equations or inequalities to describe numbers or relationships.</td>
</tr>
<tr>
<td>CC.2.2.HS.C.3</td>
<td>Write functions or sequences that model relationships between two quantities.</td>
</tr>
<tr>
<td>CC.2.2.HS.C.5</td>
<td>Construct and compare linear, quadratic, and exponential models to solve problems.</td>
</tr>
<tr>
<td>CC.2.2.HS.C.6</td>
<td>Interpret functions in terms of the situations they model.</td>
</tr>
<tr>
<td>CC.2.2.HS.C.4</td>
<td>Interpret the effects transformations have on functions and find the inverses of functions.</td>
</tr>
<tr>
<td>CC.2.2.HS.D.10</td>
<td>Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.</td>
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<td>CC.2.4.HS.B.2</td>
<td>Summarize, represent, and interpret data on two categorical and quantitative variables.</td>
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<td>CC.2.4.HS.B.3</td>
<td>Analyze linear models to make interpretations based on the data.</td>
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<tr>
<td>CC.2.4.HS.B.5</td>
<td>Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.</td>
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</table>

## Misconceptions

1. Strong correlation implies a cause-and-effect relationship between two variables.
2. A correlation coefficient of 0 implies no relationship between two variables.
3. The least squares line for predicting \( y \) from \( x \) is the same as the least squares line for predicting \( x \) from \( y \).
4. The LSRL can always be used to make predictions.
5. The LSRL will give accurate predictions as long as the data values lie within the given data set.
6. Influential observations are observations that are far away from the regression line.
7. A pattern in a residual plot indicates that a linear model is a good fit.

## Proper Conceptions

1. Correlation does not imply causation.
2. A correlation coefficient of 0 indicates that there is not a linear relationship between two variables. Some other nonlinear relationship may exist.
3. These two lines are not the same line. The independent and dependent variables must be clearly defined.
4. Extrapolation should not be used when making predictions.
5. The adequacy of the model must first be interpreted. The standard deviation about the least squares line and the coefficient of determination should both be considered before using the LSRL as a model for prediction.
6. Influential observations are those whose \( x \)-values are far away from most of the \( x \)-values in the data set.
7. A residual plot with no apparent pattern indicates that a linear model is a good fit.

## Concepts

- Pearson’s Sample Correlation Coefficient
- Population correlation coefficient
- Least squares regression
- Making predictions using

## Competencies

- Calculate and interpret the sample and population correlation coefficients
- Calculate and interpret the LSRL for bivariate data
- Use the LSRL to find predicted and residual values
- Calculating and interpreting the coefficient of determination
- Calculate and interpret the standard deviation about the LSRL
- Transform a nonlinear function
- Use power transformations

## Vocabulary

- Scatterplot
- Pearson’s sample correlation coefficient
- Principle of least squares
- Slope of least-squares line
- \( y \)-intercept of least-squares line
| the LSRL | Use logistic regression |
| Residuals | Interpret and communicate the results of statistical analyses |
| Constructing residual plots to assess the LSRL | |
| Standard deviation about the LSRL | |
| Coefficient of determination | |
| Transformations | |
| Power transformation | |
| Logistic regression | |
| Predicted (fitted) values | |
| Residuals | |
| Residual plot | |
| Residual (error) sum of squares | |
| Total sum of squares | |
| Coefficient of determination | |
| Standard deviation about the least-squares line | |
| Transformation | |
| Power transformation | |
| Logistic regression function | |

**Assessments**

**Homework** – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
**Unit Quizzes** – Quizzes will be given throughout the unit to evaluate students’ understanding of the material.
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**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction*

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**Differentiation:**

- Graphic organizers may be provided.
- Multiple concrete examples of concepts will be provided.
- Extended time may be permitted for completing projects, quizzes, or tests.
- Lessons will be presented using both visual and auditory means of communication.

**Interdisciplinary Connections:**

- Biology
- College Life
- Demography and Population Characteristics
- Environmental Science
- Food Science
- Medical Science
- Public Health and Safety
- Sports
Additional Resources (May include but are not limited to the following):
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www.collegeboard.org

Created By:
Thomas A. Seltzer
Unit Title
Unit 6: Probability

Unit Overview
Every day we make decisions based on uncertainty. The decisions we make vary from simple decisions like whether it is “worth the risk” to leave our umbrella at home based on the weather forecast to more complicated decisions such as whether it is worth the risk to undergo a certain surgical procedure. Using the ideas and methods of probability, the systematic study of uncertainty, can help us make these decisions.

Unit Essential Questions
1. What is a chance experiment?
2. What is an event?
3. How are probabilities determined?
4. What are the basic properties of probability?
5. What is conditional probability and how is it calculated?
6. What are independent events and how are they determined?
7. What does it mean to sample with and without replacement?
8. What are the general probability rules?
9. How is simulation used to estimate probabilities?

Key Understandings
1. Define and identify the key components of a chance experiment including the sample space, events, and simple events
2. Find the complement of an event and the union and intersection of two events
3. Define probability and explain the difference between the classical, subjective and relative frequency approach to determining probabilities
4. Identify the four fundamental properties of probability
5. Define and calculate conditional probabilities
6. Identify independent events and calculate their probabilities
7. Explain what it means to sample with replacement and without replacement
8. Explain and apply the general addition rule for two events, the general multiplication rule for two events, the law of total probability, and Bayes’ Rule
9. Use simulation to estimate probabilities empirically
## Focus Standards Addressed in the Unit

<table>
<thead>
<tr>
<th>CC.2.4.HS.B.6</th>
<th>Use the concepts of independence and conditional probability to interpret data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC.2.4.HS.B.7</td>
<td>Apply the rules of probability to compute probabilities of compound events in a uniform probability model.</td>
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### Misconceptions
1. Probabilities can have values greater than 1 or less than 0.
2. All probabilities are calculated the same way.
3. The subjective approach to probability is reliable.

### Proper Conceptions
1. All probability values must be between 0 and 1 inclusive.
2. There are several different ways probabilities are calculated.
3. The subjective approach to probability is highly suspect. The classical and relative frequency approaches are much more reliable for making decisions.

## Concepts
- Chance Experiments
- Sample Space
- Events / Simple Events
- Complement of an event / union and intersection of two events
- Disjoint or mutually exclusive events
- Different approaches to probability
- Basic properties of probability
- Addition rule for disjoint events
- Conditional probability
- Independent and dependent events
- Multiplication rule for k independent events
- Sampling with

## Competencies
- Identify the sample space and events of a chance experiment
- Model sample spaces using tree diagrams
- Find the complement of an event and the union and intersection of two or more events
- Identify disjoint events
- Use Venn diagrams to model the complement of an event, the intersection of two events, the union of two events and disjoint events
- Explain the different approaches to probability; classical, subjective, and relative frequency
- State the four fundamental properties of probability
- Apply various rules to calculate probabilities for a variety of situations
- Determine if events are independent or dependent.
- Explain the difference between sampling with and without replacement
- Use simulation to estimate probabilities empirically

## Vocabulary
- Chance experiment
- Sample space
- Event
- Simple event
- Events
- Complement of an event
- Union and intersection of two events
- Disjoint (mutually exclusive) events
- Conditional probability
- Independence of events E and F
- Bayes’ Rule
and without replacement
• General addition rule
• General multiplication rule
• Law of total probability
• Bayes’ Rule
• Estimating probabilities empirically

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Manufacturing and Industry
Marketing and Consumer Behavior
Medical Science
Politics and Public Policy
Psychology, Sociology and Social Issues
Public Health and Safety
Sports
Transportation

Additional Resources (May include but are not limited to the following):
Textbook Ancillary Materials
College Board AP Course Guidelines
AP Statistics / Grade 11-12 / Unit 7: Random Variables and Probability Distributions

<table>
<thead>
<tr>
<th>Subject</th>
<th>Grade</th>
<th>Unit 7: Random Variables and Probability Distributions</th>
<th>Suggested Timeline</th>
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<tbody>
<tr>
<td>Mathematics</td>
<td>11 - 12</td>
<td></td>
<td>16 Days</td>
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**Grade Level Units**
- Unit 1: The Role of Statistics and the Data Analysis Process
- Unit 2: Collecting Data Sensibly
- Unit 3: Graphical Methods for Describing Data
- Unit 4: Numerical Methods for Describing Data
- Unit 5: Summarizing Bivariate Data
- Unit 6: Probability
- **Unit 7: Random Variables and Probability Distributions**
- Unit 8: Sampling Variability and Sampling Distributions
- Unit 9: Estimation Using a Single Sample
- Unit 10: Hypothesis Testing Using a Single Sample
- Unit 11: Comparing Two Populations or Treatments
- Unit 12: The Analysis of Categorical Data and Goodness-of-Fit Tests
- Unit 13: Simple Linear Regression and Correlation: Inferential Methods
- Unit 14: Multiple Regression Analysis
- Unit 15: Analysis of Variance

**Unit Title**
Unit 7: Random Variables and Probability Distributions

**Unit Overview**
This unit focuses on the discussion of continuous and discrete random variables and the construction of probability distributions to display the possible values taken by these variables. The probability distributions model the behavior of the variables and are useful for making inferences based on sample data. Specifically the construction and interpretation of the binomial, geometric, and normal distributions are discussed.

**Unit Essential Questions**
1. What is the difference between a continuous and a discrete random variable?
2. What are the properties of a probability distribution for a discrete random variable?
3. What are the properties of a probability distribution for a continuous random variable?
4. How is the mean and standard deviation of both a discrete and a continuous random variable calculated?
5. How are the mean and variance of linear functions and linear combinations calculated?
6. What are the properties of a binomial distribution?
7. How is the mean and standard deviation of the binomial distribution calculated?
8. What are the properties of the geometric distribution?
9. What is the normal distribution and the standard normal distribution?
10. How is the table of standard normal curve areas interpreted?
11. How are probabilities for normal distributions calculated?

**Key Understandings**
1. Identify continuous and discrete random variables
2. Construct a probability distributions for discrete and continuous random variables
3. Calculate the mean and standard deviation of both a discrete and a continuous random variable
4. Calculate the mean and variance of linear functions and linear combinations
5. Identify the properties of the binomial distribution and calculate both its mean and its standard deviation
6. Identify the properties of the geometric distribution
7. Identify the properties of both normal and standard normal distributions
8. Find and interpret probabilities using normal distributions
9. Calculate and interpret z-scores
10. Construct and interpret normal probability plots
11. Use the correlation coefficient to check normality
12. Use transformations to obtain approximately normal distributions
13. Use normal distributions to approximate discrete
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<td>12. What is a z-score?</td>
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<td>13. What is a normal probability plot and how is it used?</td>
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<td>14. How is the correlation coefficient used to check normality?</td>
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<td>15. How can transformations be used to obtain approximately normal distributions?</td>
<td></td>
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<td>16. How can the normal distribution be used to approximate a discrete distribution?</td>
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**Misconceptions**

1. The conditions for a binomial distribution and a geometric distribution are the same.
2. Z-scores represent x-values.
3. When reading a standard normal table, the areas listed represent the areas under the curve to the right of the z-score.

**Proper Conceptions**

1. The conditions for a binomial distribution include a fixed number of trials while the conditions for a geometric distribution requires that trials be continued until a success is obtained.
2. Z-scores cannot be interpreted as x-values. They are standardized values that allow us to compare one normal distribution to another. They also allow us to compare an individual value to the mean value.
3. When reading a standard normal table, the areas listed represent the areas under the curve to the left of the z-score. Sometimes these areas must be subtracted to find the area between two z-scores.

**Concepts**

- Continuous & Discrete random variables
- Probability distribution for a discrete / continuous

**Competencies**

- Define and identify continuous and discrete random variables
- Construct and interpret probability distributions for discrete random variables
- Construct and interpret probability distributions for continuous random variables
- Calculate the mean and standard deviation of both a

**Vocabulary**

- Random variable
- Discrete random variable
- Continuous random variable
- Probability of a discrete random
random variable
- Mean and standard deviation of a random variable
- Linear combinations
- Binomial, Geometric and Normal Distributions
- z-scores
- Normal probability plots
- Using transformations to obtain approximately normal distributions
- Using the normal distribution to approximate a discrete distribution

discrete and a continuous random variable
- Calculate the mean and variance of linear functions and linear combinations
- Use the properties of the binomial distribution to find probabilities
- Use the properties of the geometric distribution to find probabilities
- Use the properties of both normal and standard normal distributions to find probabilities
- Calculating and interpreting probabilities using normal distributions
- Calculating and interpreting z-scores
- Constructing and interpreting normal probability plots.
- Using the correlation coefficient to check normality
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variable x
- Probability of a continuous random variable x
- Mean of a random variable x
- Standard deviation of a random variable x
- Binomial probability distribution
- Mean of a binomial random variable
- Standard deviation of a binomial random variable
- Normal distribution
- Standard normal distribution
- z-score
- Normal approximation to the binomial distribution

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Created By:
Thomas A. Seltzer
Grade Level Units
Unit 1: The Role of Statistics and the Data Analysis Process
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Unit 15: Analysis of Variance

Unit Title
Unit 8: Sampling Variability and Sampling Distributions

Unit Overview
This unit focuses on the construction of the sampling distribution of sample means and the sampling distribution of sample proportions. These distributions are used to analyze the long run behavior of the sample mean and the sample proportion when sample after sample is selected. Such distributions are useful for making statistical inferences based on sample data.

Unit Essential Questions
1. What is a statistic?
2. What is sampling variability?
3. What is a sampling distribution?
4. What are the properties of the sampling distribution of a sample mean?
5. What are the properties of the sampling distribution of a sample proportion?
6. What is the central limit theorem and how does it apply to sampling distributions?

Key Understandings
1. Explain the concept of sampling variability
2. Construct and interpret sampling distributions
3. Identify and verify the properties of the sampling distribution of a sample mean
4. Identify and verify the properties of the sampling distribution of a sample mean
5. Correctly apply the central limit theorem to sampling distributions

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Misconceptions

1. The value of a statistic calculated from a set of data and the value of a statistic calculated from a sampling distribution of a statistic have the same meaning.
2. It is necessary to calculate the statistic for all possible samples in order to describe the sampling distribution.

Proper Conceptions

1. The statistic calculated from a set of data is representative of a single data set. That is, it comes from a single sample. The statistic calculated from a sampling distribution of the statistic is the result of the sample statistic results obtained from multiple samples.
2. Patterns emerge that enable us to describe some important aspects of the sampling distributions for some statistics without having to look at all possible samples.

Concepts

- Statistic
- Sampling Distribution
- Sampling Distribution of the sample mean
- Central Limit Theorem
- Sampling Distribution of the sample proportion

Competencies

- Use sampling distributions to describe the long-run behavior of statistics
- Apply the properties of the sampling distribution of a sample mean to draw conclusions
- Apply the central limit as it applies to sampling distributions
- Apply the properties of the sampling distribution of a sample proportion to draw conclusions

Vocabulary

- Statistic
- Sampling distribution
- Sampling distribution of \( \bar{X} \)
- Central Limit Theorem
- Sampling distribution of \( \hat{p} \)

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**Created By:**
Thomas A. Seltzer
Subject: Mathematics  
Grade: 11 - 12

Unit 9: Estimation Using a Single Sample

Suggested Timeline: 10 Days

Grade Level Units
Unit 1: The Role of Statistics and the Data Analysis Process  
Unit 2: Collecting Data Sensibly  
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Unit 15: Analysis of Variance

Unit Title
Unit 9: Estimation Using a Single Sample

Unit Overview
The branch of inferential statistics is concerned with drawing inferences from sample data for the purpose of decreasing our level of uncertainty about a population characteristic such as the population mean or population proportion. Using sample data we can obtain a value that represents a plausible value for the population characteristic. We can also use sample data to determine a range of plausible values for the population characteristic. Point estimation and interval estimation are the main focus of this unit.

Unit Essential Questions
1. What is a point estimate?  
2. How do you choose a statistic for computing a point estimate?  
3. What is an unbiased statistic?  
4. What is a confidence interval?  
5. What is a confidence level?  
6. How is a large-sample confidence interval for a population proportion constructed?  
7. How is the confidence level interpreted?  
8. What is the standard error of a statistic?  
9. What is the bound on error of estimation?  
10. What is the sample size required to estimate a population proportion $p$ to within an amount $B$ with 95% confidence?  
11. How is a confidence interval for a population mean constructed?  
12. What is the difference between a one-sample $z$ confidence interval for the population mean and a one-sample $t$ confidence interval for the population mean?  

Key Understandings
1. Calculate point estimates.  
2. Determine if a statistic is biased or unbiased  
3. Construct and interpret confidence intervals  
4. Compute the sample size necessary for estimating both the population proportion and the population mean to within an amount $B$ with 95% confidence.  
5. Identify the standard error of a statistic  
6. Explain the difference between a one-sample $z$ confidence interval for the population mean and a one-sample $t$ confidence interval for the population mean.  
7. State the properties of $t$-distributions
13. What are the properties of t distributions?
14. What is the sample size required to estimate a population mean to within an amount B with 95% confidence?

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Misconceptions

1. A point estimate is a reliable estimate of a population parameter.
2. A high confidence level with a wide confidence interval provides useful information.
3. The accuracy of estimation depends on the population size.
4. The assumptions for confidence intervals are not important.

Proper Conceptions

1. The point estimate must be accompanied by a bound on error or some other measure of accuracy.
2. High confidence is not the same thing as saying we have precise information about the value of a population characteristic. The best strategy for decreasing the width of a confidence interval is to take a larger sample.
3. The accuracy of estimation depends on the sample size.
4. Assumptions and “plausibility” conditions are important and must be taken into account when constructing confidence intervals.

Concepts

- Point Estimates
- Biased and Unbiased statistics
- Confidence interval
- Confidence Level
- Large sample confidence interval for the population proportion.
- Large sample confidence interval for the population mean.
- Standard error of a statistic.
- Bound on the error of estimate.

Competencies

- Define, calculate and interpret point estimates
- Determine if a statistic is biased or unbiased
- Interpret the meaning of a confidence level
- Construct and interpret a confidence interval for the population proportion when the sample size is large
- Construct and interpret a confidence interval for the population mean when (a) the population standard deviation is known and either the sample size is large or the population distribution is normal and (b) when the population standard deviation is unknown and either the sample size is large or the population distribution is normal
- Compute the sample size necessary for estimating both the population proportion and the population mean to within an amount B with 95% confidence
- Identify the standard error of a statistic
- Explain the difference between a one-sample z confidence interval for the population mean and a one-sample t confidence interval for the population mean

Vocabulary

- Point estimate
- Unbiased statistic
- Confidence interval
- Confidence level
- Large sample confidence interval for $p$
- Sample size
- Bound on the error of estimation
- Large sample confidence intervals for $\mu$
Assessments
Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Quizzes – Quizzes will be given throughout the unit to evaluate students’ understanding of the material.
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit.
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Suggested Strategies to Support Design of Coherent Instruction
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Differentiation:
• Graphic organizers may be provided.
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• Extended time may be permitted for completing projects, quizzes, or tests.
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Interdisciplinary Connections:
College Life
Environmental Science
Medical Science
Politics and Public Policy
Public Health and Safety
Transportation

Additional Resources (May include but are not limited to the following):
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www.collegeboard.org

Created By:
Thomas A. Seltzer
Subject: Mathematics  
Grade: 11 - 12  
Unit 10: Hypothesis Testing Using a Single Sample  
Suggested Timeline: 11 Days

Grade Level Units
Unit 1: The Role of Statistics and the Data Analysis Process  
Unit 2: Collecting Data Sensibly  
Unit 3: Graphical Methods for Describing Data  
Unit 4: Numerical Methods for Describing Data  
Unit 5: Summarizing Bivariate Data  
Unit 6: Probability  
Unit 7: Random Variables and Probability Distributions  
Unit 8: Sampling Variability and Sampling Distributions  
Unit 9: Estimation Using a Single Sample  
Unit 10: Hypothesis Testing Using a Single Sample  
Unit 11: Comparing Two Populations or Treatments  
Unit 12: The Analysis of Categorical Data and Goodness-of-Fit Tests  
Unit 13: Simple Linear Regression and Correlation: Inferential Methods  
Unit 14: Multiple Regression Analysis  
Unit 15: Analysis of Variance

Unit Title  
Unit 10: Hypothesis Testing Using a Single Sample

Unit Overview
This unit focuses on the use of sample data to make a decision regarding a claim (hypothesis) about a population characteristic. The process for performing a hypothesis test about a population characteristic is outlined and the possible outcomes of a hypothesis test are discussed.

Unit Essential Questions
1. What is a hypothesis test?  
2. What is the null hypothesis?  
3. What is the alternative hypothesis?  
4. What are the two possible conclusions of a hypothesis test?  
5. What is a type I error?  
6. What is a type II error?  
7. What is a test statistic?  
8. What is a P-value?  
9. What are the steps in a hypothesis test?  
10. What is the process for performing a Large-Sample z Test for p?  
11. What is the process for performing a Large-Sample t Test for the population mean?  
12. What is the difference between using z or t for a Large-Sample Hypothesis Test for a population mean?  
13. What does it mean for the result of a hypothesis test to be statistically significant?  
14. What is the power of a test?  
15. How is the probability of a type II error calculated?

Key Understandings
1. Explain what a hypothesis test is and how it is used  
2. Correctly write hypothesis for a hypothesis test  
3. Use hypothesis tests to make decisions concerning the plausibility of a population characteristic  
4. Identify the types of errors that are present in a hypothesis test and interpret their meaning  
5. Define both test statistic and P-value and explain their role in performing a hypothesis test  
6. Outline the general steps for a hypothesis test  
7. Outline the specific procedures for performing a Large-Sample z Test for p  
8. Outline the specific procedures for performing a Large-Sample t Test for the population mean  
9. Understand the difference between the use of z or t when performing a Large-Sample Hypothesis Test for a population mean  
10. Explain the concept of statistical significance.  
11. Determine the power of a test  
12. Calculate and interpret the probability of a type II error
### Focus Standards Addressed in the Unit

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<tr>
<th>CC.2.4.HS.B.1</th>
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<td>Recognize and evaluate random processes underlying statistical experiments.</td>
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<td>CC.2.4.HS.B.5</td>
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### Misconceptions

1. Statistics can be used to write hypotheses.
2. The alternative hypothesis may contain a statement of equality.
3. A hypothesis test can prove a hypothesis.
4. A hypothesis test can be used to support the null hypothesis.
5. There are many conclusions to a hypothesis test.

### Proper Conceptions

1. Only population parameters can be used to write hypotheses.
2. The alternative hypothesis must always be stated as an inequality and may not contain a statement of equality. That is, the alternative hypothesis can only show the population parameter as being less than, greater than or not equal to a hypothesized value.
3. Hypothesis tests cannot prove a hypothesis is true. They can only be used to give supporting evidence for a claim.
4. A hypothesis test can never be used to support the null hypothesis. It can only give strong evidence that the null hypothesis is not true. Consequently if you are trying to support a claim, you must state the claim as the alternative hypothesis.
5. Only two conclusions can be reached from a hypothesis test: 1) Fail to reject the null hypothesis – you do not have enough evidence to conclude the null hypothesis is not plausible, or 2) Reject the null hypothesis in favor of the alternative hypothesis – you have sufficient evidence to conclude that the null hypothesis is not plausible and consequently you conclude the alternative hypothesis is plausible.

### Concepts

- Hypothesis test
- Null Hypothesis
- Alternative Hypothesis
- Conclusions of a Hypothesis Test
- Type I Error
- Type II Error
- Test Statistic
- P-Value
- General steps for performing a

### Competencies

- Properly write and execute hypothesis tests for both the population proportion and the population mean showing all of the steps in the hypothesis test procedure
- Correctly interpret the result of hypothesis tests to make decisions concerning the plausibility of a population proportion or a population mean
- Identify both type I and type II errors in a hypothesis test and discuss their implications
- Choose the proper test statistic (z or t) when performing a hypothesis test
- Determine whether the results of a hypothesis test are statistically significant

### Vocabulary

- Hypothesis
- Null hypothesis
- Alternative hypothesis
- Type I error
- Type II error
- Test statistic
- P-value
- Power
**Assessments**
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*Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction*

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**Interdisciplinary Connections:**
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Education and Child Development
Environmental Science
Manufacturing and Industry
Marketing and Consumer Behavior
Medical Science
Public Health and Safety
Sports
Transportation
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Created By:
Thomas A. Seltzer
AP Statistics / Grade 11-12 / Unit 11: Comparing Two Populations or Treatments

Grade Level Units
Unit 1: The Role of Statistics and the Data Analysis Process
Unit 2: Collecting Data Sensibly
Unit 3: Graphical Methods for Describing Data
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Unit 13: Simple Linear Regression and Correlation: Inferential Methods
Unit 14: Multiple Regression Analysis
Unit 15: Analysis of Variance

Unit Title
Unit 11: Comparing Two Populations or Treatments

Unit Overview
Often the reason for a statistical investigation is to compare two populations or treatments. This unit will focus on the use of specific hypothesis tests and confidence intervals for comparing two populations or treatments. The specific tests and intervals to be discussed include the two-sample t test for comparing two populations, the two-sample t test for comparing two treatments, the two-sample t confidence interval for the difference between two population or treatment means, the paired t test for comparing two population or treatment means, and the paired t confidence interval for the mean value of the difference population. Large-sample z tests for the differences between two proportions and large-sample confidence intervals for the difference between two proportions are also discussed.

Unit Essential Questions
1. What are independent samples?
2. What are the properties of the sampling distribution of the difference between two sample means?
3. What is the procedure for completing the two-sample t test for comparing two populations?
4. How do the procedures for the two-sample t test for comparing two population and the two-sample t test for comparing two treatments differ?
5. What is the procedure for constructing a two-sample confidence interval for the difference between two population or treatment means?
6. What is the procedure for completing the paired t test for comparing two population or treatment means?
7. What is the procedure for completing the paired t confidence interval for the difference between population means?
8. What are the properties of the sampling distribution of the difference of sample proportions

Key Understandings
1. Identify independent samples
2. State the properties of the sampling distribution of the difference between two sample means
3. Perform hypothesis tests using two samples
4. Construct confidence intervals for two samples
5. State the properties of the sampling distribution of the difference of sample proportions
9. What is the procedure for completing the large-sample z tests for the difference between two population proportions?

10. What is the procedure for completing the large sample z confidence interval for the difference between two population proportions?

Focus Standards Addressed in the Unit

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Misconceptions

1. The outcome of a hypothesis test can convince us that there is no difference between population means or proportions.
2. If you have complete information on the two populations, it is necessary to complete a hypothesis test to compare the populations.
3. It is always appropriate to pair samples.

Proper Conceptions

1. We cannot be convinced that there is no difference between population means or proportions since a hypothesis test can never show strong support for the null hypothesis (the statement of no difference).
2. If you have complete information, it is not appropriate to perform a hypothesis test or construct a confidence interval.
3. You must look at the samples to determine if they should be paired before performing a hypothesis test.

Concepts

- Independent samples
- Sampling distribution of the difference between two sample means.
- Two-Sample t Test for Comparing Two Populations
- Two-Sample t Test for Comparing Two Treatments
- Two-Sample t Confidence Interval for the Difference Between Two Population or Treatment Means.
- Paired samples
- Paired t Test for

Competencies

- Identify and compare independent samples
- State the properties of the sampling distribution of the difference between two sample means
- Perform two-sample t tests for comparing two populations and interpreting the results
- Perform two-sample t tests for comparing two treatments and interpreting the results
- Explain the difference between the two-sample t-test for comparing two populations and the two-sample t test for comparing two treatments
- Construct and interpret a two-sample confidence interval for the difference between two population or treatment means
- Perform paired t tests for comparing two population or treatment means and interpreting the results
- Construct and interpret a paired t confidence interval for the difference between population means
- State the properties of the sampling distribution of the difference of sample proportions
- Perform large-sample z tests for the difference between two population proportions and interpret the results
- Construct and interpret a large-sample z confidence interval for the difference between two population proportions

Vocabulary

- Independent samples
- Paired samples
- Test statistic
- Confidence interval
- Degrees of freedom
- Sample mean difference
- Standard deviation of the sample differences
- Mean value for the population of differences
- Standard deviation for the population of differences
Comparing two Population or Treatment Means

- Degrees of freedom
- Sample mean difference
- Standard deviation of the sample differences
- Mean value for the population of differences
- Standard deviation for the population of differences
- Sampling distribution of the difference of sample proportions
- Large-Sample z test for the difference between two population proportions.
- Large Sample z confidence interval for the difference between two population proportions.

Assessments

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**Interdisciplinary Connections:**
- Biology
- Education and Child Development
- Medical Science
- Politics and Public Policy
- Psychology, Sociology and Social Issues
- Sports

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**Created By:**
- Thomas A. Seltzer
## Unit Title

Unit 12: The Analysis of Categorical Data and Goodness-of-Fit Tests

## Unit Overview

This unit provides inferential methods for analyzing both univariate and bivariate categorical data sets. Specifically, the unit will discuss the process of performing a chi-square goodness-of-fit test for univariate data. It will then discuss the process of performing the chi-square test for homogeneity and the chi-square test for independence for bivariate data.

### Unit Essential Questions

1. What is a one-way frequency table?
2. What is the chi-square distribution?
3. What is the process for performing a chi-square goodness-of-fit test?
4. What is a two-way frequency table (contingency table)?
5. What are marginal totals?
6. What is the process for performing a chi-square test for homogeneity?
7. What is the process for performing a chi-square test for independence?
8. What is the difference between the 3 chi-square tests presented in this unit? When is each test used?

### Key Understandings

1. Create and interpret one-way frequency tables
2. Use the chi-square distribution to perform multiple tests

### Focus Standards Addressed in the Unit

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<thead>
<tr>
<th>Standard</th>
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Important Standards Addressed in the Unit

CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

CC.2.4.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.

Misconceptions
1. The chi-square test for homogeneity and the chi-square test for independence are the same test.
2. Using a chi-square test, we can conclude there is convincing evidence that the null hypothesis is true.
3. The assumptions of a chi-square test are just that, assumptions. They do not need to be checked since we are assuming them to be true.
4. A strong correlation between two variables indicates a cause-and-effect relationship.

Proper Conceptions
1. Though the tests have many similar steps, the chi-square test for homogeneity and the chi-square test for independence differ in both their hypotheses and their conclusions.
2. Failure to reject the null hypothesis does not “prove” that the null hypothesis is true. It simply indicates that we do not have enough evidence to conclude it was not true.
3. The assumptions for the chi-square tests must be checked or incorrect conclusions may result.
4. Strong associations between variables do not imply a causal relationship.

Concepts
- One-way frequency table
- Chi-square test statistic
- Chi-square goodness-of-fit test
- Two-way frequency table
- Contingency table
- Chi-square test for homogeneity
- Chi-square test for independence

Competencies
- Perform and interpret the results of a chi-square goodness-of-fit test
- Create and interpret two-way frequency tables
- Perform and interpret the results of a chi-square test for homogeneity
- Perform and interpret the results of a chi-square test for independence
- Choose the correct chi-square test for a given problem situation

Vocabulary
- One-way frequency table
- Chi-square test statistic
- Chi-square goodness-of-fit test
- Two-way frequency table
- Contingency table
- Chi-square test for homogeneity
- Chi-square test for independence

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Thomas A. Seltzer
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Unit 15: Analysis of Variance

Unit Title
Unit 13: Simple Linear Regression and Correlation: Inferential Methods

Unit Overview
This unit provides inferential methods for bivariate numerical data, including a confidence interval (interval estimate) for a mean y value, a prediction interval for a single y value, and a test of hypotheses regarding the extent of correlation in the entire population of the (x, y) pairs.

Unit Essential Questions
1. What is a deterministic relationship?
2. What is the simple linear regression model?
3. What are the basic assumptions of the simple linear regression model?
4. How is the slope and y-intercept of a population regression line determined?
5. What is the interpretation of a + bx* where x* is a specified value of the predictor variable x?
6. How is the variance and standard deviation of the population regression line determined?
7. What are the properties of the sampling distribution of the slope of the least squares line?
8. How is the estimated standard deviation of the slope of the least squares line calculated?
9. How is the confidence interval for the slope of the population regression line constructed?
10. What is the process for performing hypotheses tests concerning the slope of the population regression line?
11. What is the process for performing the model utility

Key Understandings
1. Define what is meant by deterministic relationships.
2. Use the simple linear regression model
3. Construct and interpret a confidence interval for the slope of the population regression line
4. Perform hypotheses tests using the simple linear regression model
12. How is residual analysis used to check model adequacy?
13. What are the properties of the sampling distribution of \( a + bx \) for a fixed \( x \) value?
14. How is the estimated standard deviation of the statistic \( a + bx \) * calculated?
15. How is a confidence interval for \( a + bx \) * constructed?
16. How is the prediction interval for a single \( y \) value constructed?
17. What are the steps for the test for independence in a bivariate normal population?

Focus Standards Addressed in the Unit

CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

CC.2.4.HS.B.3 Analyze linear models to make interpretations based on the data.

CC.2.4.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.

Important Standards Addressed in the Unit

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CC.2.4.HS.B.3 Analyze linear models to make interpretations based on the data.

CC.2.4.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.

Misconceptions

1. All regression lines are useful and can be used to make predictions
2. The method by which the data for the regression line is collected is not important
3. Assumptions are not important

Proper Conceptions

1. It does not make sense to use a regression line as the basis for making inferences about a population if there is no convincing evidence of a useful linear relationship between the two variables under study.
2. Inferential methods used in this unit are only appropriate if the data used to construct the regression line come from a random sample.
3. Assumptions should be checked before performing computations.

Concepts

- Simple linear regression
- Estimating the population regression line
- Estimating the variance and standard deviation of the population regression line
- Inferences about the slope of the population regression line

Competencies

- State the basic assumptions of the simple linear regression model
- Find point estimates of the slope and \( y \)-intercept of the population regression line and interpret the meaning of \( a+bx \) * where \( x \) * denotes a specified value of the predictor variable \( x \)
- Calculate and interpret the estimated variance and estimated standard deviation of the population regression line
- State the properties of the sampling distribution of \( b \)
- Construct and interpret a confidence interval for the slope of the population regression line
- Perform and interpret hypothesis tests concerning the slope of the population regression line

Vocabulary

- Simple linear regression model
- Estimated regression line
- Point estimate of the standard deviation
- Estimated standard deviation of the statistic \( b \)
- Confidence interval
- Model utility test
- Residual analysis
- Standardized
• Checking model adequacy
• Inferences based on the estimated regression line
• Inferences about the population correlation coefficient

• Perform and interpret the model utility test for simple linear regression
• Use residual analysis to check for model adequacy
• Stating the properties of the sampling distribution of $a+bx$ for a fixed $x$ value
• Construct and interpret a confidence interval for a mean $y$ value
• Construct and interpret a prediction interval for a single $y$ value
• Perform a test for independence in a bivariate normal population and interpret the results

Assessments
Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
Unit Quizzes – Quizzes will be given throughout the unit to evaluate students’ understanding of the material.
Unit Test – A unit test will be given at the end of the unit to evaluate students’ overall understanding of the unit.
Unit Notebook – Students’ notebooks will be checked at the end of each instructional unit to reinforce organizational skills
Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment

Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson’s Framework for Teaching: Domain 3 Instruction
3a – An assignment sheet for the unit will communicate the timeline for the unit as well as the assignments and assessments to be completed.
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3e – Pacing of lessons may be adjusted and additional examples may be given as necessary to enhance student understanding.

Differentiation:
• Graphic organizers may be provided.
• Multiple concrete examples of concepts will be provided.
• Extended time may be permitted for completing projects, quizzes, or tests.
• Lessons will be presented using both visual and auditory means of communication.

Interdisciplinary Connections:
Biology
Demography and Population Characteristics
Medical Science
Physical Sciences
Politics and Public Policy
Sports

Additional Resources (May include but are not limited to the following):
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
AP Statistics Test Preparation Workbooks
www.collegeboard.org

Created By:
Thomas A. Seltzer
Subject: Mathematics  
Grade: 11 - 12  

Unit 14: Multiple Regression Analysis  

Suggested Timeline: 6 Days

Grade Level Units
Unit 1: The Role of Statistics and the Data Analysis Process
Unit 2: Collecting Data Sensibly
Unit 3: Graphical Methods for Describing Data
Unit 4: Numerical Methods for Describing Data
Unit 5: Summarizing Bivariate Data
Unit 6: Probability
Unit 7: Random Variables and Probability Distributions
Unit 8: Sampling Variability and Sampling Distributions
Unit 9: Estimation Using a Single Sample
Unit 10: Hypothesis Testing Using a Single Sample
Unit 11: Comparing Two Populations or Treatments
Unit 12: The Analysis of Categorical Data and Goodness-of-Fit Tests
Unit 13: Simple Linear Regression and Correlation: Inferential Methods
Unit 14: Multiple Regression Analysis
Unit 15: Analysis of Variance

Unit Title
Unit 14: Multiple Regression Analysis

Unit Overview
This unit extends the methods used in the previous unit for linear regression to multiple regression models which include two or more predictor variables. The calculations required to perform fit a multiple regression model are more involved than those used for simple linear regression and consequently require the use of technology.

Unit Essential Questions
1. What is a general additive multiple regression model?
2. What is a kth-degree polynomial regression model?
3. What is meant by interaction between two variables?
4. What is an indicator (dummy) variable?
5. How is a multiple regression model fit and assessed?
6. What is the F distribution?
7. What are the steps for performing the F-test for model utility?

Key Understandings
1. Explain how multiple regression models are fit and assessed
2. Perform and interpret F-tests for model utility

Focus Standards Addressed in the Unit

CC.2.4.HS.B.2  
Summarize, represent, and interpret data on two categorical and quantitative variables.

CC.2.4.HS.B.3  
Analyze linear models to make interpretations based on the data.

CC.2.4.HS.B.5  
Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
Important Standards Addressed in the Unit

<table>
<thead>
<tr>
<th>Standard</th>
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<tbody>
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<td>CC.2.4.HS.B.2</td>
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<td>CC.2.4.HS.B.5</td>
<td>Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.</td>
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Misconceptions
1. A useful model is one that results in both a large $R^2$ value and a small $s_e$ value. These two conditions can be achieved by fitting a model that contains a large number of predictors.

Proper Conceptions
1. Using a large number of predictors may be successful in explaining $y$ variation in the data, but it almost always specifies a relationship that cannot be generalized to the population and that may be unrealistic and difficult to interpret. The goal is a simple model with few predictors that can easily be interpreted.

Concepts
- Multiple regression models
- Polynomial regression
- Interaction between variables
- Qualitative predictor variables
- Fitting a model and assessing its utility
- F distributions
- The F test for model utility

Competencies
- Use the general additive multiple regression model
- Use a kth-degree polynomial regression model
- Explain interaction between two variables
- Identify “indicator” or “dummy” variables
- Fit and assess multiple regression models
- Describe the F distribution
- Perform and interpret F-tests for model utility

Vocabulary
- Additive multiple regression model
- Estimated regression function
- Coefficient of multiple determination
- Adjusted $R^2$
- F distribution
- F test
- Test statistic

Assessments
Homework – Problems assigned and reviewed daily to reinforce and enhance students’ understanding of concepts
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Suggested Strategies to Support Design of Coherent Instruction

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Differentiation:
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Interdisciplinary Connections:
Business and Economics
College Life
Education and Child Development
Environmental Science
Manufacturing and Industry
Medical Science
Physical Sciences
Transportation

Additional Resources (May include but are not limited to the following):
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Created By:
Thomas A. Seltzer
Unit Title
Unit 15: Analysis of Variance

Unit Overview
This unit introduces the concept of single-factor analysis of variance (ANOVA) and the procedures for analyzing these types of problems. The objective of ANOVA problems is to determine whether the means for more than two populations or treatments are equal.

Unit Essential Questions
1. What is a single-factor analysis of variance (ANOVA) problem?
2. What are the assumptions for ANOVA?
3. What are the steps for performing the single-factor ANOVA F test?
4. What is a multiple comparisons procedure?
5. What is the Tukey-Kramer multiple comparisons procedure?
6. How are the results of the Tukey-Kramer procedure summarized?

Key Understandings
1. Identify problems that involve single-factor analysis of variance
2. Perform ANOVA F-tests
3. Perform the Tukey-Kramer multiple comparisons procedure

Focus Standards Addressed in the Unit

**CC.2.4.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

**CC.2.4.HS.B.4** Recognize and evaluate random processes underlying statistical experiments.

**CC.2.4.HS.B.5** Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
Important Standards Addressed in the Unit

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<tr>
<td>CC.2.4.HS.B.1</td>
<td>Summarize, represent, and interpret data on a single count or measurement variable.</td>
</tr>
<tr>
<td>CC.2.4.HS.B.4</td>
<td>Recognize and evaluate random processes underlying statistical experiments.</td>
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<tr>
<td>CC.2.4.HS.B.5</td>
<td>Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.</td>
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Misconceptions
1. When comparing different groups to see if there is any difference in their means, you can simply construct boxplots for each group and compare the locations of each of the means.

Proper Conceptions
1. Since there can be substantial overlap in the boxplots, a formal test procedure (ANOVA) must be used to test if the means are the same for each of the different groups.

Concepts
- Single-factor ANOVA
- The F test
- Multiple Comparisons
- The Tukey-Kramer Multiple Comparison Procedure

Competencies
- Identify problems that involve ANOVA
- State the assumptions for ANOVA
- Perform all steps necessary for the single-factor ANOVA F test and interpret the results
- Describe what is meant by a multiple comparisons procedure
- Perform the Tukey-Kramer multiple comparisons procedure, summarize and interpret the results

Vocabulary
- Single-factor analysis of variance (ANOVA)
- Treatment sum of squares
- Error sum of squares
- Mean square
- Test statistic
- Total sum of squares
- Tukey-Kramer multiple comparison procedure

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Agriculture
Business and Economics
Manufacturing and Industry
Medical Science

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